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Direct imaging rapidly-rotating non-Kerr black holes

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Recently, two of us have argued that non-Kerr black holes in gravity theories different from General Relativity may have a topologically non-trivial event horizon. More precisely, the spatial topology of the horizon of non-rotating and slow-rotating objects would be a 2-sphere, like in Kerr space-time, while it would change above a critical value of the spin parameter. When the topology of the horizon changes, the black hole central singularity shows up. The accretion process from a thin disk can potentially overspin these black holes and induce the topology transition, violating the Weak Cosmic Censorship Conjecture. If the astrophysical black hole candidates are not the black holes predicted by General Relativity, we might have the quite unique opportunity to see their central region, where classical physics breaks down and quantum gravity effects should appear. Even if the quantum gravity region turned out to be extremely small, at the level of the Planck scale, the size of its apparent image would be finite and potentially observable with future facilities.

I. INTRODUCTION

General Relativity (GR) is our current theory of gravity and up to now there is no clear observational evidence in disagreement with its predictions. Nevertheless, the theory has been tested only for weak gravitational fields, where $g_{tt} \approx -(1+\phi)$ and $|\phi| \ll 1$ [1]. There are instead physically interesting situations in which GR breaks down and predicts space-time singularities and regions with closed time-like curves. Here, (unknown) quantum gravity effects are thought to become important. According to the Weak Cosmic Censorship Conjecture (WCCC) [2], these quantum gravity regions are always hidden behind an event horizon and we would have no chances to observe them. However, the assumption of the WCCC is essentially motivated to assure the validity of classical gravity in any astrophysical situation, with the guess that new physics can never be seen by a distant observer. For the time being, there is no fundamental principle requiring that [3].

In 4-dimensional GR, uncharged rotating black holes (BHs) are described by the Kerr solution, which is completely specified by two parameters, the mass M and the spin parameter a. The condition for the existence of the event horizon is $a \le M$, while for $a > M$ there is no BH but a naked singularity, which is forbidden by the WCCC[1]. While it is not yet clear if naked singularities can be created in Nature [4], any attempt to make a star collapse with $a > M$ [5], as well as to overspin an already existing Kerr BH and get a naked singularity [6], seems to be doomed to fail. In Ref. [7], two of us considered the loop-inspired BHs proposed in [8] and the non-GR BHs introduced in [9]. In addition to M and a, these space-times have at least one more parameter: it can be seen as a "deformation parameter" and it produces deviations from the Kerr geometry. It was shown that the spatial topology of the horizon of these objects is a 2-sphere (like for a Kerr BH) in the non-rotating and slow-rotating case, while it changes above a critical value of a/M. Our conjecture is that such a phenomenon may be common for non-Kerr BHs. The basic mechanism is the following. A Kerr BH with $a/M < 1$ has an outer horizon with radius r_+ and an inner horizon with radius r_-. As a/M increases, r_+ decreases and r_- increases. When $a/M = 1$, there is only one horizon (extremal black hole with $r_+ = r_-$) and, for $a/M > 1$, there is no horizon. For the BHs studied in [7], the outer and the inner horizons have not the same shape. So, when a/M increases, the two horizons still approach each other, but eventually merge forming a single horizon with non-trivial topology. After the topology transition, the BH central singularity (or the high curvature region if the singularity is solved, as it occurs at least for some loop-inspired BHs) is not hidden behind the horizon any more. Interestingly, such a rapidly rotating BHs can be easily created: it is just necessary a thin accretion disk [7].

II. BLACK HOLES IN ALTERNATIVE THEORIES OF GRAVITY

In this paper, we consider the BH metric proposed in Ref. [9]. However, as emphasized in [7], the phenomenon we are going to study should depend only marginally on this choice and be common to rapidly-rotating non-Kerr

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[1] Throughout this paper, we use the convention $a = |a|$ and units in which $G_N = c = 1$.

BHs. Together with the loop-inspired BH solution in [8], they are the only known 4D electrically-neutral non-Kerr metrics in analytical form valid beyond the non-rotating and slow-rotating approximation. The non-zero metric coefficients, in Boyer-Lindquist coordinates, are [9][2]

$$\begin{aligned} g_{tt} &= -\left(1-\frac{2Mr}{\rho^2}\right)(1+h)\,, \\ g_{t\phi} &= -\frac{2aMr\sin^2\theta}{\rho^2}(1+h)\,, \\ g_{\phi\phi} &= \sin^2\theta\left[r^2+a^2+\frac{2a^2Mr\sin^2\theta}{\rho^2}\right]+ \\ &\quad +\frac{a^2(\rho^2+2Mr)\sin^4\theta}{\rho^2}h\,, \\ g_{rr} &= \frac{\rho^2(1+h)}{\Delta+a^2h\sin^2\theta} \quad , \quad g_{\theta\theta}=\rho^2\,, \end{aligned} \tag{2.1}$$

where

$$\begin{aligned} \rho^2 &= r^2+a^2\cos^2\theta\,, \\ \Delta &= r^2-2Mr+a^2\,, \\ h &= \sum_{k=0}^{\infty}\left(\epsilon_{2k}+\frac{Mr}{\rho^2}\epsilon_{2k+1}\right)\left(\frac{M^2}{\rho^2}\right)^k\,. \end{aligned} \tag{2.2}$$

Assuming that the function h is infinitely time differentiable, the metric has an infinite number of free parameters ϵ_i and the Kerr solution is recovered when all these parameters are set to zero. As shown in [9], $\epsilon_0=\epsilon_1=0$, in order to recover the correct Newtonian limit, while Solar System experiments constrain ϵ_2 at the level of 10^{-4}. For the sake of simplicity, in the rest of the paper we restrict our attention to the case in which ϵ_3 is the only deformation parameter and $\epsilon_i=0$ for $i\neq 3$. In this case, as discussed in [7], current estimates of the mean radiative efficiency of AGN seem to require $-1.1<\epsilon_3<25$, but such a bound has to be taken with caution.

R_{QG}/M	$\Delta y/M$
0.1	0.444
0.01	0.412
0.001	0.410

TABLE I. Vertical size of the primary image at $x=0$ of the quantum gravity region of a black hole with $a/M=1.18$ and $\epsilon_3=-1.0$. The viewing angle of the distant observer is $i=5^\circ$. R_{QG} is the radius in Boyer-Lindquist coordinates of the quantum gravity region. See text for details.

[2] As suggested in Ref. [10], a metric of this kind can be solution of a particular non-local generalization of the Einstein's equations:

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=8\pi G_N\mathcal{O}(\Box/\Lambda^2)T_{\mu\nu}\,,$$

where $\mathcal{O}(\Box/\Lambda^2)$ is a generic non-local function of the covariant D'Alembertian operator and Λ is the energy scale of the modified gravity.

III. DIRECT IMAGING BLACK HOLES WITH NON-TRIVIAL TOPOLOGY

The geometry around a BH can be probed by observing the direct image of its accretion flow, as already explored in [11]. For a review on current and near future capabilities of Very Long Baseline Interferometry (VLBI) experiments, see e.g. [12] and references therein. If the gas of accretion is optically thin (which is always possible at sufficiently high frequencies) and geometrically thick, one sees the "shadow"; that is, a dark area over a brighter background [13]. While the intensity map of the image strongly depends on the specific accretion model, i.e. on the configuration of the system and on complicated astrophysical processes, the contour of the shadow is determined only by the geometry of the space-time: basically, it is the photon capture surface as seen by a distant observer. For rotating BHs, the shape of the shadow is not symmetric with respect to the rotation axis, because the capture radius for corotating photons is smaller than the one for counterrotating photons. The effect is more evident for fast-rotating BHs and observers with a viewing angle $i=90^\circ$, while it disappears for non-rotating BHs or observers along the rotation axis.

In this section, we consider two specific cases; this is enough to illustrate the qualitative features of the BHs described by the metric (2.1). The first BH has $a/M=1.18$ and $\epsilon_3=-1.0$. As $\epsilon_3<0$, the object is more oblate than a Kerr BH. $a/M=1.18$ corresponds to the equilibrium value in the case the BH is accreting from a thin disk on the equatorial plane, see Table II in [7]. The equilibrium spin parameter is indeed always larger than M when a compact object is more oblate than a Kerr BH [14]. The horizon of this BH is defined by $g^{rr}=0$; that is:

$$\Delta+a^2h\sin^2\theta=0 \tag{3.1}$$

and it is shown in the left panel of Fig. 1. As already pointed out in Ref. [7], rapidly-rotating oblate BHs have a shape similar to a donut. However, in the specific case of the metric (2.1), there is no central hole: the event horizon extends up to $r=0$, where there is a naked singularity. As second example, we consider a BH with $a/M=0.87$ and $\epsilon_3=1.0$. As in the previous case, this value of a/M corresponds to the one of equilibrium when the BH is accreting from a thin disk on the equatorial plane. When $\epsilon_3>0$, the object is more prolate than a Kerr BH and its horizon, still given by Eq. (3.1), is formed by two disconnected horizons, each of which with spatial topology of a 2-sphere, as shown in the right panel of Fig. 1.

Fig. 2 shows the contour of the shadow (black curve) and the apparent images of the central singularity (in red) of the first BH with $a/M = 1.18$ and $\epsilon_3 = -1.0$. The viewing angle of the distant observer is $i = 5°$ (left panel), 45° (central panel), and 85° (right panel). The BH shadow is computed by considering all the photons crossing perpendicularly the screen of the observer and numerically integrating backward in time the geodesic equations for the metric (2.1). The light rays crossing the event horizon are inside the shadow, while the ones coming from infinity are outside. The resolution of the screen of the observer is 0.001 M. More details about the computational procedure can be found in previous papers [11]. The quantum gravity region has been modeled as a ball of constant radius (in Boyer-Lindquist coordinates) $R_{QG} = 0.1\ M$. The choice of R_{QG} is arbitrary here. However, as R_{QG} decreases, the apparent size of the quantum gravity region seems to approach a constant value, as shown in Tab. I. *Even if the size of the quantum gravity region is extremely small, say of order of the Planck scale, the size of its apparent image is finite. Such a result is quite interesting and it may open a new way to probe the Planck scale with astrophysical observations.*

Fig. 3 shows the trajectories on the xz-plane of some light rays coming from the quantum gravity region. The primary image is formed by light rays traveling along almost-straight paths, while the trajectories of the light rays responsible for the other images are strongly bent by the gravitational field of the BH.

Fig. 4 shows the apparent image of the second BH with $a/M = 0.87$ and $\epsilon_3 = 1.0$. The shape of this shadow has a very peculiar structure on the side of the corotating photons ($x_{obs} > 0$ in the central and right panel of Fig. 4) due to the existence of two disconnected event horizons; a similar feature has never be found for BHs and other compact objects with trivial topology [11]. It is thus an observational signature of this BHs with $\epsilon_3 > 0$ and high spin parameter. Unlike the images in Fig. 2, here the distant observer cannot see the central singularity at $r = 0$, for any value of the viewing angle. That is due to the presence of the two disconnected horizons, above and below the equatorial plane.

IV. DISCUSSION

As argued in Ref. [7], rapidly-rotating non-Kerr BHs may have a topologically non-trivial event horizon. In this work, we have studied how astrophysical observations can test this scenario. Interestingly, when these BHs are overspun and the topology of the horizon changes, the BH central singularity shows up and, at least in some cases, it can be seen by a distant observer. In the previous section, we have shown only the apparent images of two specific cases, but this is enough to figure out all the qualitative features. When $\epsilon_3 < 0$ and $a/M > 1$, we can usually see the central region at $r = 0$, but its apparent size decreases/increases for lower/higher values of a/M. For $\epsilon_3 = -1.0$, a BH with $a/M = 1.18$ is thus the most favorable case in a realistic context, because BHs with higher a/M may not be created in Nature. When $\epsilon_3 > 0$, the central region of the BH cannot be seen for spin parameters not exceeding the equilibrium value of a BH accreting from a thin disk. For higher values of the spin parameter, the central region may be seen by a distant observer, but these objects can unlikely be created, as it seems to occur for a Kerr naked singularity. On the other hand, the peculiar structure of the shape of the shadow due to the existence of two disconnected horizons is common to all these BHs. However, it is more evident for high values of a/M and ϵ_3.

To test this scenario, it may not be strictly necessary to observe the exact shape of the BH shadow. When $\epsilon_3 < 0$, the detection of the radiation coming from the quantum gravity region may be enough if it has peculiar properties. Assuming that such a quantum gravity region can emit some form of radiation, the latter has likely very high energies, as the central region should be governed by Planck scale physics. For instance, we could try to use these BHs with a visible central singularity to explain the correlation observed by the Pierre Auger experiment between ultra high energy cosmic rays (UHECRs) and active galactic nuclei (AGN) [15]. The observed cosmic rays with energies above $6 \cdot 10^{19}$ eV are definitively difficult to explain with standard acceleration mechanisms, while here they might be produced by Planck-energy particles emitted from the quantum gravity region of AGN, which are indeed thought to harbor at their center very rapidly-rotating super-massive BHs.

The prediction of the BH apparent image does not require the knowledge of specific features of the quantum gravity region. We have just assumed it may emit electromagnetic radiation. It is however definitively intriguing to think about the possible properties of this region. For the time being, we do not have any reliable theory of quantum gravity, and therefore we cannot know what really happens to the classical singularity. However, there are a few scenarios proposed in the literature. For instance, in the BHs inspired by Loop Quantum Gravity [8], as well as in the ones motivated by non-commutative geometries [16], the central singularity turns out to be replaced by a Planck length size region violating the weak energy condition. The size of the quantum gravity region is therefore independent of the BH mass, while the energy density changes. At least in the case of the BHs in [16], the shape of the quantum gravity region is similar to a ring.

Lastly, let us notice that even in GR, when the spacetime has more than four dimensions, BHs can have event horizons with topology different from the one of the sphere. Thus, non-trivial event horizons might be a signature of higher dimensions as well.

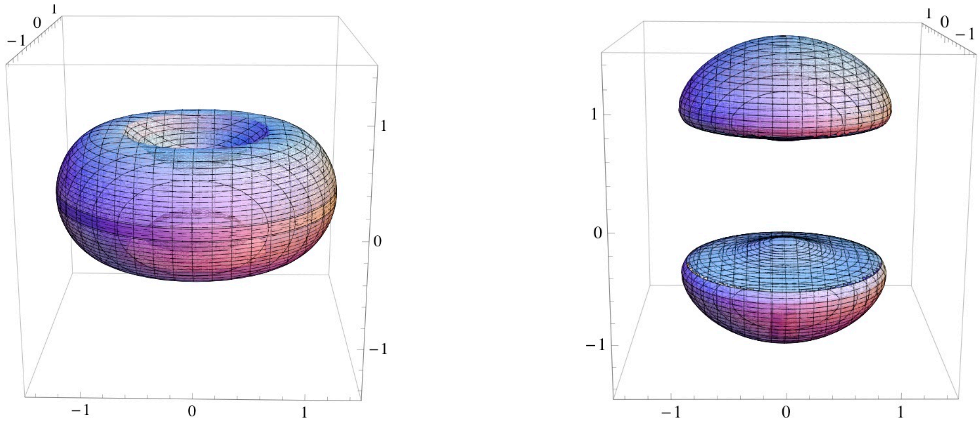

FIG. 1. Event horizon of the two black holes studied in Sec. III. Left panel: black hole with $a/M = 1.18$ and $\epsilon_3 = -1.0$. Right panel: black hole with $a/M = 0.87$ and $\epsilon_3 = 1.0$. Axes in units $M = 1$.

V. CONCLUSIONS

It is thought that the final product of the gravitational collapse is a Kerr black hole and astronomers have discovered several good astrophysical candidates. While there is some indirect evidence suggesting that these objects have really an event horizon [17], we do not yet know if the space-time around them is described by the Kerr geometry. Recently, there has been an increasing interest in the possibility of testing the Kerr black hole hypothesis with present and near future experiments [11, 18].

As argued in Ref. [7], black holes with generic deformations from the predictions of General Relativity may change the topology of the horizon above a critical value of the spin parameter. The accretion process from a thin disk can potentially overspin these black holes and induce the topology transition, which makes the phenomenon astrophysically interesting. In this paper, we have discussed how such a possibility can be tested. We have studied the propagation of light rays in these space-times and we have computed the direct image of these objects. Our results are summarized in Fig. 2 (for a black hole more oblate than a Kerr one) and Fig. 4 (for a black hole more prolate than a Kerr one). As the contour of the shadow (the black curves in Figs. 2 and 4) is determined by the geometry of the space-time, an accurate observation of the direct image of rapidly-rotating black hole candidates may test this scenario. Moreover, we have found the more exciting possibility that a distant observer may see the central region of these black holes, where classical physics breaks down and quantum gravity effects should appear. The apparent images of this quantum gravity region, here simply modeled as a luminous ball of constant radius, are shown in red in Fig. 2.

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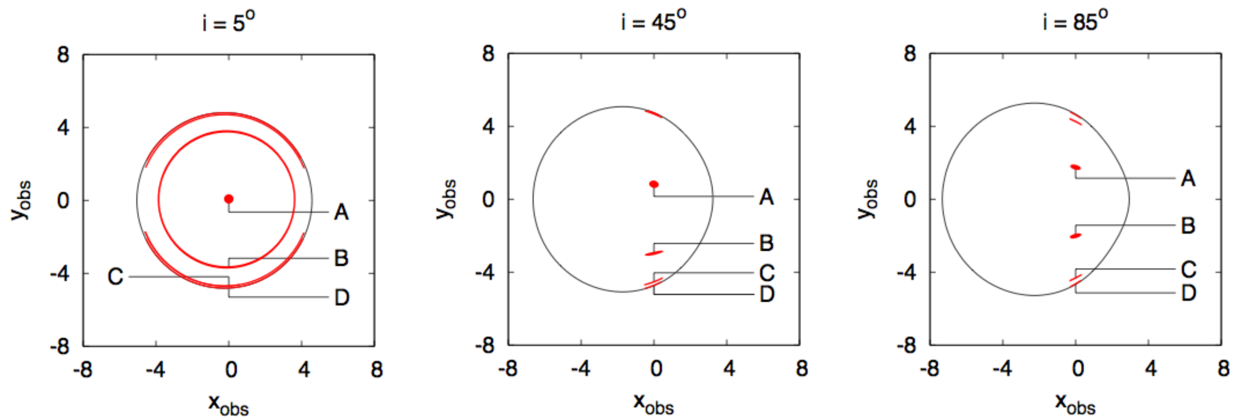

FIG. 2. Apparent image of the black hole with $a/M = 1.18$ and $\epsilon_3 = -1.0$ for a distant observer with viewing angle $i = 5^\circ$ (left panel), 45° (central panel), and 85° (right panel). The black curve is the contour of the black hole shadow. The red area is the image of the quantum gravity region. The trajectories of the light rays A, B, C, and D around the black hole are shown in Fig. 3. Axes in units $M = 1$.

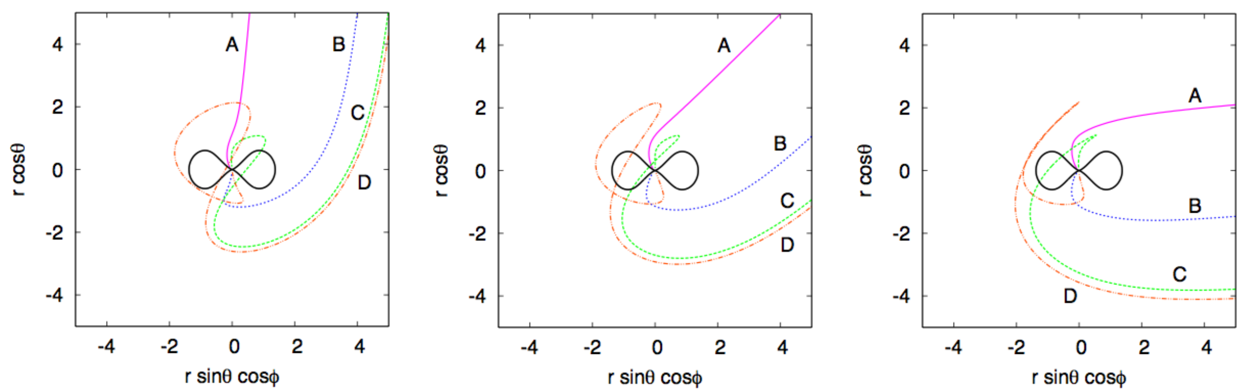

FIG. 3. Trajectories on the xz-plane of the light rays A, B, C, and D of Fig. 2, for a distant observer with viewing angle $i = 5^\circ$ (left panel), 45° (central panel), and 85° (right panel). The primary image of the quantum gravity region is formed by light rays like A, while the light rays like B, C, and D are responsible for the multiple images. Axes in units $M = 1$.

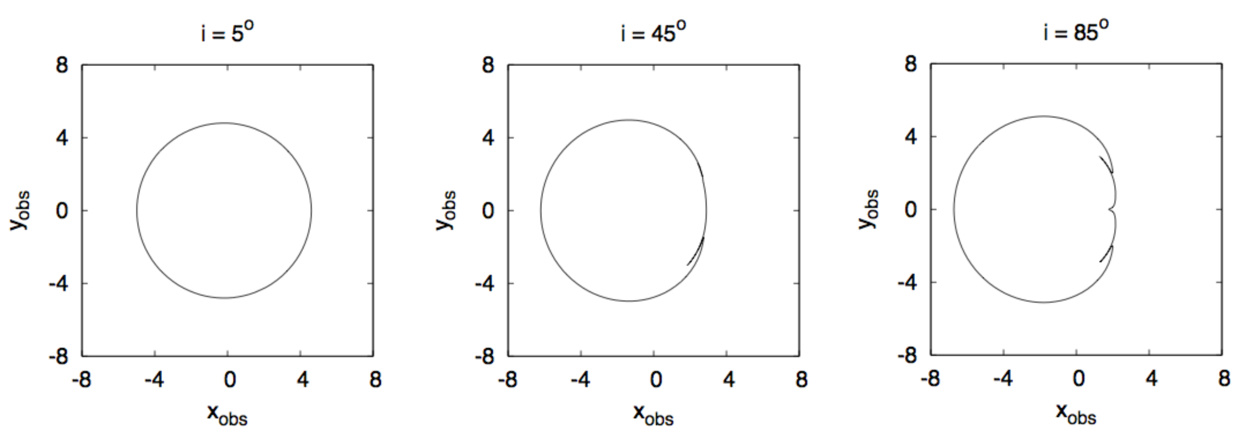

FIG. 4. As in Fig. 2, for the case of the black hole with $a/M = 0.87$ and $\epsilon_3 = 1.0$. Here, the distant observer cannot see the quantum gravity region.

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